Birthday paradox

If there are possible outcomes, then trying times will most likely result in 2 of the outcomes being the same.

Let’s say you try times.

Note that uniform distribution is not required. In fact, If it is not uniform, it is more likely to have 2 overlapping results.

Example: 4 distinct digits

Given 2 integers and , find a multiple of with length such that it consists of no more than 4 distinct digits.

is too large, cannot use dp!

Observation: If you have 2 base 10 numbers with digits consisting with only ‘0’ and ‘1’, the difference between them will only consist of digits ‘0’, ‘1’, ‘8’ and ‘9’.

Example:

Therefore, If you have 2 such numbers with length , and their remainders mod are the same, the difference will satisfy the conditions of the problem.

What if the difference has leading zeros? You can just add zeros at the back until the length is because it will still be divisible by .

How to generate 2 numbers with the same remainder? We can use the birthday paradox!

Generate numbers of length consisting of digits ‘0’ and ‘1’. Then, it is virtually certain that 2 of them will have the same remainder mod .

Time complexity:

Blogewoosh #6 trick

There are hidden integers . You are allowed to ask at most queries. For each query, you can choose 2 integers and and ask if . Find the maximum .

Solution 1:

Do independent binary searches for each .

Time complexity:

Solution 2:

Before doing binary search on , ask if it is strictly greater than the maximum of what we have seen before. If it isn’t, there is no point and we go to the next .

However, if the hidden array is increasing, we will do binary search times and it would still be very slow. To tackle this problem, we can consider them in a random order! On average, the maximum will be located in the middle of the array, and the maximum before that will also be located in the middle of the subarray, and so on. Therefore, the average number of binary searches is .

Time complexity: